Optimal production zone

Given the following production function with 2 productive factors K and L:

$$F(K, L) = 4L^2K - 0.1L^3K$$

where K is fixed at 5, obtain:

1. Functions of: Short-term total product (TP), marginal product (MP), and average product (AP).
2. Calculate the amount of the variable factor when the Law of Diminishing Marginal Returns begins to apply.
3. Graph and highlight the Optimal Production Zone.

Solution

1. With $K = 5$, we have $F(L) = 20L^2 - 0.5L^3$, which is the short-term total product. The marginal product is:

$$MP = F'(L) = 40L - 1.5L^2$$

$$AP = F(L)/L = 20L - 0.5L^2$$

2. To determine where the law of diminishing returns applies, we need to calculate the point where MP has a maximum:

$$MP'(L) = 40 - 3L = 0$$

$$L = 40/3$$

Beyond $L = 40/3$, the marginal product starts to decline.

3. The average product is:

$$AP = F(L)/L = 20L - 0.5L^2$$

The average product reaches its maximum when:

$$AP'(L) = 20 - L = 0$$

$$L = 20$$

On the other hand, the marginal product is:

$$MP = 40L - 1.5L^2$$

And this reaches zero when:

$$MP'(L) = 40 - 3L = 0$$

$$L = 26.667$$

The production zones are as follows:

$$\text{Zone 1: } 0 \leq L < 20$$

$$\text{Zone 2: } 20 \leq L \leq 26.667$$

$$\text{Zone 3: } L > 26.667$$

The optimal production zone is Zone 2.

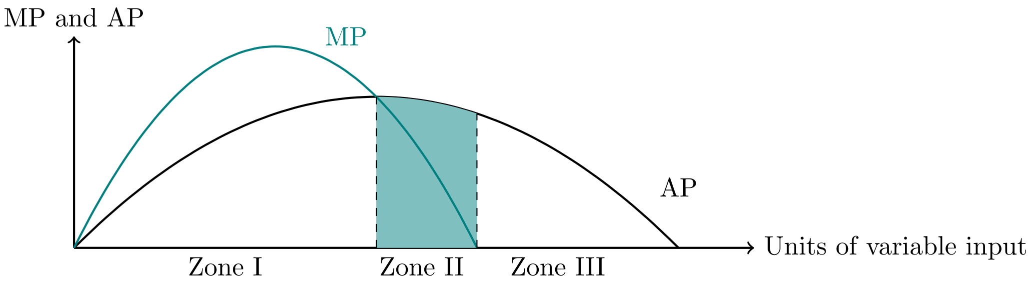

Figure 1: Marginal Product and Average Product

The optimal production zone begins when the average product is at its maximum (maximizing average efficiency) and ends when the marginal product is zero (maximum total production without decline). This range ensures the most efficient and productive use of the variable factor.